6, 8, 10, and 12. Despite having large range of variation of θ, the experimental results are seen to be very well correlated by the suggested parameters. The straightline, fitted on the basis of the least-square principle and shown in Fig. 3, corresponds to a power-law trajectory

$$X/DR = 1.75(Y/DR)^{0.38} \tag{12}$$

The exponent found here in Eq. (12) for inclined jets is the same as that found earlier in Eq. (6) for normal jets—representing a special case of inclined jets with $X = x$ and $Y = y$. The discrepancy between the coefficients is apparently due to the use of different devices to produce the jets.[1] A uniform initial velocity across the jet was assured by Pratte and Baines by using an orifice, whereas a pipe-flow (parabolic) velocity distribution is introduced by Platten and Keffer, who used a long tube for their means of jet injection. Such differences should not affect the exponent, however, although they can be expected to influence the coefficient of the power-law profile.

Conclusions

It has been shown here that use of a skewed coordinate system, aligning the coordinate axes with the jet and the stream velocity vectors, can greatly simplify the description of the trajectories of turbulent jets discharging at various angles of inclination into a uniform cross stream and having various ratios between the initial jet velocity and the stream velocity. By aid of this skewed coordinate system and on the basis of dimensional and similarity considerations, the centerline trajectories of such jets appear to be very well correlated by a simple power law. The centerline trajectory and induced pressure field of the deflected jets also can be determined by integrating numerically the momentum equation from the given initial condition at the jet exit and with the entrainment coefficient and the crossflow drag coefficient chosen from the test data. Such calculations certainly are important for determining the pressure field induced by the deflected jet, but the present formula should provide a convenient and sufficient estimation of the centerline trajectory of the deflected jets.

References

[1] Platten, J. L. and Keffer, J. F., "Deflected Turbulent Jet Flows," *Journal of Applied Mechanics*, Vol. 38, No. 4, 1971, pp. 756–758.

[2] Keffer, J. F. and Baines, W. D., "The Round Turbulent Jet in a Crosswind," *Journal of Fluid Mechanics*, Vol. 15, Pt. 4, 1963, pp. 481–496.

[3] Pratte, B. D. and Baines, W. D., "Profiles of the Round Turbulent Jet in a Cross Flow," *Journal of the Hydraulics Division, ASCE*, Vol. 92, No. HY 6, 1967, pp. 53–64.

[4] Albertson, M. L., Dai, Y. B., Jensen, R. A., and Rouse, H., "Diffusion of Submerged Jets," *Transactions of the American Society of Civil Engineers*, Vol. 115, 1950, pp. 639–664.

Unsteady Boundary-Layer Flow of Power Law Fluids

SREEDHAN ROY*
University of Delhi, Delhi, India

Nomenclature

A, B = dimensional constants, Eqs. (3) and (4)
f = nondimensional velocity
k = coefficient in constitutive equation for power law fluids
m = constant, Eq. (4)
n = power law fluid index
t = time
u = velocity in boundary layer
U = velocity just outside boundary layer
y = distance normal to surface
α = positive constant, Eq. (3)
ν = kinematic viscosity of Newtonian fluids
η = similarity variable
ρ = density of fluid

Received May 15, 1973.
Index categories: Boundary Layers and Convective Heat Transfer—Laminar; Nonsteady Aerodynamics.
* Pool Officer, Department of Mathematics.

THE purpose of this Note is to show that two-dimensional flows past surfaces immersed in non-Newtonian power law fluids, which start moving impulsively with velocities proportional to t^α, admit of similarity solutions. The previous analytical studies that have come to the notice of the author include the works of Bird,[1] Wells,[2] Rott[3] and Chen and Wollersheim.[4] In particular, Chen and Wollersheim studied the case $\alpha = 0$. Roy[5] performed a perturbation analysis assuming the fluid to be slightly non-Newtonian and showed that the results thus obtained compared very favorably with those predicted by Chen and Wollersheim within the range $0.5 \leqq n \leqq 1.5$.

Mathematically speaking, we have to solve the boundary-layer equation

$$\partial u/\partial t = \partial U/\partial t + (k/\rho)(\partial/\partial y)(\partial u/\partial y)^n \tag{1}$$

subject to the boundary conditions

$$\begin{aligned} &u = 0, \quad y = 0, \quad t > 0 \\ &u \to U \quad \text{as} \quad y \to \infty, \quad t > 0 \\ &\text{and at} \quad t = 0_+, \quad y > 0 \end{aligned} \tag{2}$$

Assume

$$U = At^\alpha \tag{3}$$

where A and α are constants.

These equations have similarity solutions given by

$$\left.\begin{aligned} \eta = y/Bt^m, \quad u = Uf(\eta) \\ m = (1/n+1)\{(n-1)\alpha+1\} \\ B = \{2n(n+1)KA^{n-1}/\rho[(n-1)\alpha+1]\}^{1/(n+1)} \end{aligned}\right\} \tag{4}$$

where f satisfies the equation

$$\left.\begin{aligned} (m/n)(d/d\eta)(f')^n + 2m\eta f' + 2\alpha(1-f) = 0 \\ f(0) = 0, \quad f(\infty) = 1 \end{aligned}\right\} \tag{5}$$

It can be easily seen that for $\alpha = 0$ we attain the equations of Chen and Wollersheim. Also for $n = 1$ the similarity variable and similarity equations become

$$\left.\begin{aligned} \eta = y/2(\nu t)^{1/2} \\ \text{and} \\ f'' + 2\eta f' + 4\alpha(1-f) = 0 \\ f(0) = 0, \quad f(\infty) = 1 \end{aligned}\right\} \tag{6}$$

Equations (6) have been obtained and solved by Roy[6] for different values of α.

The numerical solutions of Eq. (5) are presented in Table 1 for $\alpha = \frac{1}{2}$ and 1. It is observed that the values of $f'(0)$ at first increase with n up to $n = 1$ and then decrease. Similar trends about $n = 1$ have been widely exhibited by Shah's calculations for steady flows of power law fluids.[7]

Table 1 Values of $f'(0)$ for different values of α

n	$\alpha = 1/2$	$\alpha = 1$
0.25	1.6452	1.9395
0.50	1.7142	2.1085
0.75	1.7553	2.2096
1.00	1.7725	2.2568
1.25	1.7710	2.2553
1.50	1.7579	2.2322
1.75	1.7378	2.2120
2.00	1.7133	2.0081

References

[1] Bird, R. B., "Unsteady Pseudoplastic Flow Near a Moving Wall," *AIChE Journal*, Vol. 5, No. 4, Dec. 1959, p. 565.

[2] Wells, C. S., Jr., "Unsteady Boundary Layer Flow of a Non-Newtonian Fluid on a Flat Plate," *AIAA Journal*, Vol. 2, No. 5, May 1964, pp. 951–952.

[3] Rott, W., "Laminare Strömung dilatanter Flüssigkeiten in der Nähe einer plötzlich in Bewegung gesetzen Platte," *Chemical Engineering Science*, Vol. 26, 1971, pp. 739–742.

[4] Chen, T. Y. W. and Wollersheim, D. E., "Solution of Unsteady Flow of Power Law Fluids," *AIAA Journal*, Vol. 10, No. 5, May 1972, pp. 689–691.

[5] Roy, S., "Contribution in Unsteady Flow of Power Law Fluids," *AIAA Journal*, Vol. 11, No. 2, Feb. 1973, pp. 238–239.

[6] Roy, S., "Diffusion in a Time-Dependent Three-Dimensional Boundary Layer," *Indian Journal of Physics*, Vol. 46, 1972, pp. 370–378.

[7] Shah, M. J., Ph.D. thesis, 1961, Univ. of California at Berkeley, Berkeley, Calif.

Stability Analysis of Cylinders with Circular Cutouts

B. O. Almroth,* F. A. Brogan,† and M. B. Marlowe‡
Lockheed Palo Alto Research Laboratory, Palo Alto, Calif.

Nomenclature

X, θ	= shell surface coordinates
X, Y, Z	= Cartesian coordinates
R	= cylinder radius
h	= cylinder thickness
a	= cutout dimension (see Fig. 3)
F	= function [Eq. (2)]
w	= lateral displacement
N	= axial stress resultant
P	= applied axial load
N_0	= critical value of stress resultant
P_0	= critical value of axial load
$\bar{w}$	= w/h
$\bar{N}$	= N/N_0
$\bar{P}$	= P/P_0
α	= $a/(Rh)^{1/2}$

An analysis is presented for the stability of axially compressed cylinders with circular cutouts. The numerical analysis was based on an extension of the finite-difference method which removes the requirements that displacement components be defined in the directions of the grid lines. The results from the nonlinear analysis are found to be in good agreement with previously available experimental results.

For shells of general shape and for shells of revolution whose symmetry has been destroyed by cutouts or local stiffening, loss of stability usually occurs by collapse at a limit load point. In such cases, stability analyses based upon the assumption of bifurcation from a linear prestress may be of little practical significance[1]; hence, to establish a meaningful collapse load for the shell, we must solve numerically a set of nonlinear partial differential equations. However, this approach leads to a massive computational effort, and only recently has it been possible to obtain solutions in agreement with test results for the collapse of shells with cutouts.

Received May 10, 1973. This work was sponsored in part by the Lockheed Independent Research Program and in part under Contract NAS1-10843 with NASA/Langley Research Center.

Index category: Structural Stability Analysis.

* Senior Staff Scientist. Member AIAA.

† Research Scientist.

‡ Research Scientist. Member AIAA.

Although significant progress has been made in numerical techniques, the primary reason for the recent expansion of structural analysis capabilities is the rapid increases in computer technology. Indeed, the work reported here could not have been performed without modern high speed scientific computers such as the Control Data Corporation CDC 6600 and CDC 7600.

A collapse analysis for axially compressed cylindrical shells with rectangular cutouts was first presented in 1970 (Ref. 2). Results for similar, but thinner, cylinders were compared with experimental results in Ref. 3. It was found in this investigation that the influence of a cutout on the collapse load could be established with good accuracy by use of the STAGS computer code. This code is based upon the finite-difference energy method discussed in Refs. 4 and 5. The version of STAGS with which the results of Ref. 3 were obtained[6] is too restricted to handle efficiently shells with nonrectangular cutouts; hence, extensive modifications to the code were made before it was applied to shells with circular cutouts.

In the finite-difference energy method, as applied to shell analysis, the shell surface is divided into several area elements. At one point within each of these area elements, the displacements and their derivatives are expressed in terms of the displacement components at a number of points that may be selected independently. We shall here refer to the latter as grid points and to the points at which the energy is expressed as integration points. Through numerical integration over the integration points, the total potential energy of the system is expressed in terms of the discrete values of the displacement components at the grid points. Equilibrium configurations are found through minimization of the potential energy with respect to these free parameters.

The relations by which the displacements and their derivatives are expressed at the integration points in terms of the displacements at the grid points are referred to as finite-difference expressions. Traditionally, finite-difference methods for solution of the partial differential equations governing the behavior of thin shells have been applied in a rather restricted way. Although the method has been presented in text books in a more general form,[7] applications generally have been restricted to cases in which the grid is generated on a rectangular mapping of the shell surface, and grid and integration points are identical, being defined at intersections of coordinate lines.

The essence of the generalization presented here is that the directions of the displacement components need not coincide with the direction of the grid lines. There are no restrictions on the procedure that generates the grid points on the shell surface, but for practical reasons, it should provide a suitable numbering system for the grid points. Similar extensions have been suggested previously and applied in linear analysis.[8,9] In both these cases, the finite-difference approximations were derived by use of Green's Theorem, while in the present analysis they are based on a Taylor series expansion.

A suitable grid for a cylinder with a circular cutout can be generated by the functions

$$X = R \sin [F(\cos \theta)/R]$$
$$Y = R \cos [F(\cos \theta)/R] \qquad (1)$$
$$Z = F \sin \theta$$

where R is the radius of the cylinder and X, Y, Z are the Cartesian coordinates of a point on the cylinder (Z is measured in the axial direction). The surface coordinates are denoted by θ and x and $F(x)$ is defined by

$$F = \{[(a(1-x)+\tfrac{1}{2}\pi Rx) \sin \theta]^{2/(1-x)} + [(a(1-x)+Lx) \cos \theta]^{2/(1-x)}\}^{(1-x)/(2)} \qquad (2)$$

where a is the radius of the cutout and L is the half length of the cylinder. A computer generated picture of $\frac{1}{8}$ of a cylinder with its surface grid is shown in Fig. 1.

For cylinders with rectangular cutouts, it was shown in Ref. 3 that theoretical results could be obtained which were in very good agreement with results from experimental analysis. The